SCALAR DISSIPATION RATE MODELLING IN TURBULENT JETS USING FIRST-ORDER MODELS

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Abstract

The Transport of a scalar quantity in a turbulent round jet is addressed by testing and applying the non-equal scales model based on the two-Scale-Direct-Interaction-Approximation (TSDIA) of Yoshizawa for three first order models. In this way the assumption of equal time scales for mechanical and scalar turbulence, leading to a constant turbulent Schmidt number is no longer needed. It is shown that the trends, which are observed in the experiments, are reproduced qualitatively by the non-equal scales model.

Keywords: Scalar; Jet; Equal scales; Non-equal scales; TSDIA; Schmidt number.

1. Introduction

The jet discussed in the present study is a turbulent round jet exiting at high velocity from a round nozzle into free air. The variable density effects considered here are generated by the mixing of two gases of different density. The complexity of variable density arises because of the strong coupling between dynamic and scalar turbulent fields. The scalar can be the mass fraction of a chemical species or the temperature. The scalar dissipation rate signifies the destruction of turbulent scalar fluctuation by the action of the molecular diffusivity or conductivity and is the scalar analogue of the turbulent kinetic energy dissipation rate.

From measurements and calculations of turbulent jets and diffusion flames presented in the literature, it can be concluded that mechanical and scalar scales are not equal. The measured Prandtl number in the heated rectangular jet of Sarh\textsuperscript{[1]} was not constant, neither was the mechanical to scalar time scale ratio found in the helium-air jet of Panchapakeasan and Lumeley\textsuperscript{[2]}. Drake et al.\textsuperscript{[3]} argued that a constant turbulent Schmidt number would not be a useful concept in turbulent flames. Dibble et al.\textsuperscript{[4]} calculated the time scale ratio in a turbulent diffusion flame using a second-order closure method and find out that ratio is not constant. Lastly, Lucas\textsuperscript{[5]} and Pietri et al.\textsuperscript{[6]} found a varying mechanical to scalar time scale ratio in their experiment on axisymmetric jet.

A transport equation for the scalar dissipation rate must be constructed to overcome the assumption of equal-scales for scalar and mechanical turbulence. In the literature, most equations for the scalar dissipation rate are within the framework of second order closures. However, Yoshizawa\textsuperscript{[7]} has given a consistent derivation for this model equation within the first order closure approach with his Two-Scale-Direct-Interaction-Approximation (TSDIA).

In this paper, the non-equal scales model that was introduced, for three first order models, k-ε, RNG (Renormalisation Group) and MTS (Multiple Time Scales), was applied to investigate its performance in a turbulent round helium jet submerged in free air and measured by Lucas\textsuperscript{[5]}. In the following section, a description of the scalar dissipation rate models is introduced. The scalar dissipation rate is computed using the equal scales model and the Yoshizawa one. In the third section, we present the results of the numerical model using these different models compared with experimental results found in the literature. Finally, we present the discussion and conclusions.

2. Computational approach

The turbulent flow is modeled using Favre averaged quantities. Three first order turbulence models are used to describe the velocity turbulent field. The equations and constants of these models are described in the work of Kim\textsuperscript{[8]} (MTS), Sanders et al.\textsuperscript{[9]} (k-ε) and Yakhot et al.\textsuperscript{[10]} (RNG).

The classical algebraic equal-scales of scalar dissipation rate models $\dot{\varepsilon}_s$ are based on the assumption that the time and length scales of mechanical and scalar turbulence are equal. The time and length scales for mechanical turbulence are $\tau_u \sim k/\varepsilon$ and $l_u \sim k^{3/2}/\varepsilon$, respectively, where $k$ is the turbulent kinetic energy and $\varepsilon$ is its dissipation rate. On dimensional grounds, the time...
and length scales for scalar turbulence are \( \tau_s \sim g \varepsilon / \varepsilon_g \) and \( l_s \sim g^{3/2} \varepsilon^{1/2} / \varepsilon_g^{3/2} \), respectively, where \( g \) is the scalar variance. Setting \( \tau_u = \tau_s \) or \( l_u = l_s \) gives the equal-scales model, \( \varepsilon_g = C_g \varepsilon g / k \), where the coefficient \( C_g \) is empirical and usually taken to be equal to 2.

In the framework of first order turbulence models, the assumption of equal length scales leads to fixed turbulent Schmidt numbers. The turbulent Schmidt number, \( S_{ct} \), is defined as the ratio of the eddy viscosity \( \varepsilon \nu / k \) and the eddy diffusivity \( g^{2/3} \varepsilon / \varepsilon_g^{2/3} \). This expression leads to a relationship between \( S_{ct} \) and \( R_t \):

\[
S_{ct} \sim R_t^{2/3}.
\]

The model discussed in this work is based on the two-scale direct interaction approximation model of Yoshizawa [7] in which a scalar transport model that does not assume equal length scales and fixed turbulent Schmidt number is used. The equation for the scalar dissipation rate is obtained from an expansion of the equation for \( g \) [7].

\[
\begin{align*}
D_t \varepsilon_g + g^{5/2} \varepsilon_g^{-7/2} \varepsilon_g^{1/2} & \left( \frac{D g}{D t} \right) \\
- C_{lg} \varepsilon_g^{5/2} \varepsilon_g^{-7/2} \varepsilon_g^{1/2} & \left( \frac{D g}{D t} \right) \\
+ C_{lg} \varepsilon_g^{5/2} \varepsilon_g^{-7/2} \varepsilon_g^{1/2} & \left( \frac{D g}{D t} \right)
\end{align*}
\]  

A model equation for \( \varepsilon_g \) can be obtained by demanding transferability, which means that any model based on \( g, \varepsilon_g \) should be transferable to a model based on \( g, l_g \). This is also the case in the standard k-\( \varepsilon \) model, where in principle any combination \( (k, \varepsilon, l_u) \) can be chosen. Transferability requires an algebraic relation between \( g, \varepsilon_g \) and \( l_g \) and therefore the equation \( l_s \sim g^{3/2} \varepsilon^{1/2} / \varepsilon_g^{3/2} \) must hold. From Eq. (1) the scalar dissipation equation can then be derived.

Yoshizawa [7] using TSDIA, derived a model equation for \( \varepsilon_g \) given by

\[
\frac{D \varepsilon_g}{D t} = \varepsilon_g \left( \lambda_1 \frac{D g}{D t} + \lambda_2 \frac{D \varepsilon}{D t} \right)
\]

The particular form of Eq. (2) allows an analytical solution based on a separation of variables, which gives the expression

\[
\varepsilon_g = \phi \varepsilon_g^{\lambda_1} \varepsilon^{\lambda_2}
\]

with \( \phi \) a dimensional reference value:

\[
\phi = \frac{\varepsilon_{g0}}{g_0^{\lambda_1} \varepsilon_0^{\lambda_2}},
\]

in which the subscript 0 indicates a reference point in the flow. The dimension of the coefficient \( \phi \) is completely determined by the values of \( \lambda_1 \) and \( \lambda_2 \). By considering similarity behavior of the dependent variables and momentum conservation in a round jet, the relation between the constants should be \( \lambda_1 + 2\lambda_2 = 2 \). Using \( \lambda_1 = 1 \) and \( \lambda_2 = 1/2 \), the coefficient \( \phi \) should have the dimension of \( v^{-1/2} \), so \( m^{-1}s^{1/2} \). The value of the coefficient \( \phi \) used in this study is \( \phi = 5 m^{-1}s^{1/2} \).

A very important feature of Eq. (3) is the absence of the turbulent kinetic energy \( k \), compared with the equal-scales version where \( \varepsilon_g = C_g \varepsilon g / k \).

For calculation, the obtained equations are solved using a parabolic finite volume code. No transformation of the radial direction is employed; this means that the grid expands in the radial direction to allow the jet to expand. Note that this formulation is different from the parabolic algorithm used in the paper of Gazzah et al. [11]. 80 grid points in the radial direction and an axial forward step size of 0.01 times the local jet half width are used. The boundary conditions at the nozzle are those for fully developed pipe flow. The experimental results used for comparison with the numerical predictions are those of the air jet submerged in the free air of Lucas [5], with a corresponding density ratio of \( S_\rho = 0.14 \).

3. Results and discussions

The aim of these calculations is to investigate the development of variable density turbulent round jets emitted in a free air stream. This secondary air flow must remain sufficiently weak to avoid any radical change in the development of the jet, but must be of sufficient velocity to prevent recirculation zones from occurring. This problem is normally avoided when the Craya-Curtet parameter for variable density flows is maintained above 0.8, irrespective of the gas considered [12]. Calculations of the jets are presented using several of the models listed above. Both algebraic models are used since it is
interesting to compare these models which are easily implemented.

The axial distributions of the average mixture fraction obtained by the different models, compared with the experimental results of Lucas [5], for equal and non-equal scales models, are shown on figures 1 and 2 respectively. It is noted that all models behave similarly as to the prediction of the axial mixture fraction, and present good agreement with the experiment. The scalar fluctuation intensities are shown on figures 3 and 4. It is clear that all models give good and similar predictions of this parameter in comparison with the experimental results globally. However, non of the models is able to retrieve the constant value of this parameter in the potential core of the jet which is feature by the experiment.

The scalar fluctuation intensities normalized by the axial average mixture fraction are presented in figures 5 and 6 for equal and non-equal scales models, respectively. In both cases, it is noticed that all models predict a very rapid increase of this parameter with very similar slopes, before attaining a maximum value at about x/D=8. This maximum is flowed by a slight decrease of this parameter, after which all models tend to a constant asymptotic value of about 0.25 ~ 0.30 for x/D>20. This value is also well featured by the experiment in the mixing region, although the later decrease of the experimental values is probably associated with experimental errors.

The general behavior of these last axial distributions was also observed by Pitts [13] in the case of an isothermal jet of a gas mixing into another gas of a different molecular weight. An asymptotic value of 0.23 was obtained, although this value did not show any dependence on the density ratio of the two gases. Several other works showed the existence of this asymptotic value, such as Sarh [1] for a rectangular jet, however its value is contraversed and ranges from 0.18 to 0.26.

![Figure 1: Centerline values of the mixture fraction : Equal scales.](image1)

![Figure 2: Centerline values of the mixture fraction : Non-equal scales.](image2)
Figure 3: Centerline values of the scalar fluctuation intensities: Equal scales.

Figure 4: Centerline values of the scalar fluctuation intensities: Non-equal scales.

Figure 5: Centerline values of the scalar fluctuation intensities: Equal scales.
Figure 6: Centerline values of the scalar fluctuation intensities: Non-equal scales.

The axial distributions of the scalar dissipation rate for the equal and non-equal scales models are shown on figures 7 and 8, respectively. Here again, a similar behavior is obtained all the models, except for the MTS model which predicted a higher maximum value which is obtained earlier in the up-stream position.

Figure 7: Centerline of scalar dissipation rate: Equal scales.

Figure 8: Centerline of scalar dissipation rate: Non-equal scales.

In figure 9 radial profiles of the turbulent Schmidt number versus the radial distance normalized by the local jet scalar half-width (Ls) are plotted. The predicted turbulent Schmidt
number is compared with the measurement of Pietri et al. [6] and Chevray [14]. Qualitative agreement is found in the sense that both model and experiment show a same evolution. Also recent measurements (Panchapekesan and Lumley [2]) of $R_c$ show a similar trend (figure 10).

![Figure 9](image9.png)

**Figure 9:** Radials profiles of the turbulent Schmidt number at $x/D=10$: Non-equal scales.

![Figure 10](image10.png)

**Figure 10:** Radials profiles of the time scale ratio at $x/D=50$: Non-equal scales.

Although the non-equal scales model gives promising results there are some aspects that need further discussion.

The constant $\phi$ of Eq. (3) is dimensional and therefore cannot be universal. It would be best to relate it to a boundary condition but both at a radial boundary of the jet and at the nozzle $g$ and $\varepsilon_g$ are 0. Furthermore, at the radial boundary the energy dissipation vanishes as well, which makes the evaluation of $\phi$ impossible. The origin of this problem is that the theory of TSDIA is valid only for high Reynolds numbers where an inertial range for scalar fluctuations exists. At both aforementioned boundaries this is not the case. Therefore $\phi$ has to be related to an interior point in the flow.

A point needing further elaboration is the apparent geometry dependence of $\lambda_1$ and $\lambda_2$ as mentioned above. Both constants are non-dimensional and in the spirit of the TSDIA they are geometry independent because the inertial range forms for the scalar and mechanical energy spectrum, which they are based on, are geometry independent. The only solution which has emerged during this study is the notion that the non-equal scales model, in the form it is presented and used, has a limited range of validity with respect to the turbulence dynamics. Firstly the idea is that it is only valid for regions in the flow where an inertial range exists. The second idea is that the model could be only valid for small scales up to the energy containing range, i.e., valid for the inertial range and below. In this case another model, which necessarily would be an empirical model again, should be used for the large energy containing scales where production of turbulence...
takes place. The present model for the scalar dissipation, however, contains production terms due to scalar gradients as well as velocity gradients and extends into the energy containing range. An alternative would be the redefinition of the equation for the variance $g$ which should be associated with the transfer scales such as in equations of multiple time scales (MTS). In this way production terms which act in the large scales do not appear. Finally, with respect to this matter it is interesting to note that in early papers which were published on the theory of TSDIA [7] the equations were assumed to be valid up to the grid scales in the computational flow domain, which means that it was a sub grid model for LES.

4. Conclusion

Turbulent round jets have been investigated theoretically and numerically within the framework of the first order turbulence models. A turbulent transport model of the scalar dissipation rate has been proposed which is based on different scales for mechanical and scalar turbulence: the non-equal scales model based on the two-point TSDIA theory of Yoshizawa [7]. In results, these models are able to qualitatively predict the general behavior of variable density turbulent round jets and show good agreement with the experimental results. Predictions of the turbulent Schmidt number and the mechanical to scalar scale ratio showed a good qualitative agreement with data from experiments.

References