ELASTIC ELECTRON HELIUM SCATTERING SPECTRA ASSISTED BY A CO₂ LASER FIELD

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Abstract

We have studied theoretically the electron spectra in the elastic scattering of a fast electrons (E_i = 9.5 eV) by helium atoms in the presence of a CO₂ monochromatic laser field and in the presence of a CO₂ bichromatic laser field. The former is linearly polarized, and has moderate intensity (I=10^8 Wcm^-2) with photon energy \( \hbar \omega = 0.117 \) eV. The latter has the same polarization and has two components of frequencies \( \omega \) and \( 2\omega \) which are out of phase by an arbitrary angle \( \varphi \). Our numerical results (in the Born approximation) show that the scattered signal is sensitive to the collision geometry. Otherwise it is easily controllable by the phase variations, especially at the quadrature of the phase (\( \varphi = \pi/2 \)) the signal in the presence of the CO₂ monochromatic laser can be reproduced by a CO₂ bichromatic laser field with less intensity what would be a great advantage in laser spectroscopy.

Keywords: Electron, He-atom, Scattering geometry, Monochromatic laser, Bichromatic laser, Phase control.

1. Introduction

Electron-atom (molecule) collision in the presence of a laser field has long been a topic of considerable interest for both theorists since Bunkin and Fedorov [1] to Sadeghpour et al. [2] and experimentalists since Weingartshofer et al. [3] to Wallbank and Holmes [4], not only because of the importance of these processes in applied areas such as plasma heating, the development of powerful high frequency (XUV and X-ray) lasers, but also in view of their interest in fundamental atomic (molecular) collision theory.

Atoms and molecules in intense laser fields exhibit new properties which have been discovered via the study of multiphoton processes.

The exchange of one or more photons between the electron-atom system and the laser has been observed in laser-assisted elastic or inelastic electron-atom collisions. In previous works [5, 6] we studied the influence of a laser field on the dynamics of \((e-He)\) collisions and found that the scattered laser-assisted signal was strongly dependent on the dressing of the target by the laser field. In the present work we are interested in collision geometries effects on the laser-assisted electron helium scattering.

Three well known scattering geometries will be used. The first one (G1) was used in the experiments of Weingartshofer et al. [3]. In it the polarization vector \( \epsilon \) of the CO₂ monochromatic laser field makes an angle of 38° with the incident electron wave vector \( k \).

The two others geometries were used in many theoretical papers [7-9]. They will be designed here by the Schrödinger equation

\[
\frac{\partial}{\partial t}\chi_k(r,t) = \frac{i}{2}\left(\mathbf{p} - A(t)\right)^2\chi_k(r,t) .
\]

The non-relativistic motion of the unbound electron in the bichromatic field can be described in the velocity gauge by the Schrödinger equation

\[
\Psi_k(r,t) = (2\pi)^{-3/2} \exp[i(\mathbf{k}\cdot\mathbf{r} - k\alpha_1\sin(\omega t) - \mathbf{k}\cdot\mathbf{\alpha}_2\sin(2\alpha t + \varphi) - E_k t/\hbar)] ,
\]

with \( \mathbf{k} \) is the electron wave vector and \( E_k \) is its kinetic energy. \( \alpha_1 = E_1/\omega^2 \) and \( \alpha_2 = E_2/4\omega^2 \) are the amplitude oscillations that a classical electron would have in the bichromatic laser field. The Volkov states

\[
E(t) = \mathbf{e}_1 E_1 \sin(\omega t) + \mathbf{e}_2 E_2 \sin(2\omega t + \varphi) .
\]
the Bohr frequency and  and  are the number of the atom,  is the target state of energy for the excited states. In our expression \[10\].

The Bessel function \( J_n \) is given by the relations

\[
J_n(x) = -\frac{i}{\pi} \int_0^{\infty} \cos(x \sin \vartheta) \sin^n \vartheta \, d\vartheta,
\]

with \( R = \sum_{j=1}^{Z} r_j \), \( X \) denotes the ensemble of the target electrons coordinates \( r_1, r_2, \ldots, r_Z \), and \( Z \) is the atomic number of the atom.  is the target state of energy ,  is the Bohr frequency and  and  are the dipole-coupling matrix elements given by

\[
M_{\nu \nu', \nu} = \left\{ \Psi' \left| E, \varepsilon, \mathbf{R} \right| \Psi \right\},
\]

and \( M_{n \nu, \nu} = \left\{ \Psi' \left| E, \varepsilon, \mathbf{R} \right| \Psi \right\}. \]

The differential scattering cross section of the electron-helium atom in the presence of a bichromatic laser field can be evaluated by the expression [5]

\[
\frac{d \sigma}{d \Omega} = \left( k_f \left| (l) / k_i \right| f_{el}^{1, l} - g_{el}^{1, l} \right)^2,
\]

where \( f_{el}^{1, l} \) is the direct amplitude given by

\[
f_{el}^{1, l} (\Delta) = B_1 (a_1, a_2, \varphi) f_{el}^{1, l} (\Delta)
\]

\[
+ \frac{4 \alpha}{\Delta} (w^2 - \omega^2) J_1 (a_1, a_2, \varphi) E_{\varepsilon_i} \cdot \nabla \Delta < \Psi_\Delta | \Psi_\nu >
\]

\[
+ \frac{4 \alpha}{\Delta} (w^2 - \omega^2) J_1 (a_1, a_2, \varphi) E_{\varepsilon_i} \cdot \nabla \Delta < \Psi_\Delta | \Psi_\nu >.
\]

Here, \( \Delta (\Delta, X) \) represents the direct potential [7], \( f_{el}^{1, l} (\Delta, X) \) is the field-free first Born amplitude for the elastic collision and \( \Delta = \mathbf{k}_i - \mathbf{k}_f \) is the momentum transfer.

The generalized Bessel function \( B_1 (a_1, a_2, \varphi) \) has the expression [10].

\[
B_1 (a_1, a_2, \varphi) = \sum_{m=-\infty}^{\infty} J_m (a_2) \left( J_{m-1, 2m} (a_1) \right) e^{-im\varphi}.
\]

The Bessel function \( B_1 (a_1, a_2, \varphi) \) and \( B_2 (a_1, a_2, \varphi) \) are given by the relations

\[
B_1 (a_1, a_2, \varphi) = \frac{1}{2} \sum_{m=-\infty}^{\infty} J_m (a_2) \left( J_{m+1, 2m} (a_1) \right) e^{-im\varphi},
\]

and

\[
B_2 (a_1, a_2, \varphi) = \frac{1}{2} \sum_{m=-\infty}^{\infty} J_m (a_2) \left( \exp \varphi J_{-m, 2m} (a_1) \right)
\]

\[
- e^{-i\varphi} J_{-m, 2m} (a_1) \right) e^{-im\varphi},
\]

with the following expressions of their arguments

\( a_1 = \Delta \alpha_1 \) and \( a_2 = \Delta \alpha_2 \).

The exchange amplitude \( g_{el}^{1, l} (\Delta) \) is given by

\[
g_{el}^{1, l} (\Delta) = B_1 (a_1, a_2, \varphi) g_{el}^{1, l} (\Delta),
\]

where \( g_{el}^{1, l} (\Delta) \) is the Ochkur amplitude [11].

The monochromatic case is deduced by putting \( E_2 = 0 \). The differential scattering cross section of the electron-helium atom in the presence of a monochromatic laser field is evaluated by the expression

\[
\frac{d \sigma}{d \Omega} = \left( k_f \left| (l) / k_i \right| f_{el}^{1, l} - g_{el}^{1, l} \right)^2,
\]

with \( f_{el}^{1, l} (\Delta) = J_1 (a_1) f_{el}^{1, l} (\Delta)
\]

\[
+ \frac{4 \alpha}{\Delta} (w^2 - \omega^2) J_1 (a_1, a_2, \varphi) E_{\varepsilon_i} \cdot \nabla \Delta < \Psi_\Delta | \Psi_\nu >.
\]

3. Numerical results and discussions

We shall consider in the calculations \( E_{i=1} = E_2 \) and \( \varepsilon_1 = \varepsilon_2 \) and we shall use three scattering geometries. The first one, (G1), was used in [12]. In the second, (G2), the momentum transfer is parallel to the laser field and in the third, (G3), the momentum transfer is perpendicular to the laser field, these geometries are shown in the Figs. 1a and 1b.

![Figure 1a](image)

**Figure 1a**: Scattering geometry (G1): \( k_i, k_f \) are the initial and final momenta of the incident and scattered electron, \( \theta \) is the polar angle. The helium gas beam is perpendicular to the scattering plane, \( \varepsilon \) is the polarization vector of the electric laser field which makes an angle of 30° with the incident electron wave vector [12].

3.1. Dependence of electron spectra on collision geometry

We have used for the helium atom the undressed ground-state wave function and an average excitation energy \( \overline{\omega} = 1.15 \) a.u. for the excited states. In our spectra we have plotted the laser-assisted signal \( \frac{d \sigma_{el}^{1, l} / d \Omega}{d \sigma_{el}^{1, l} / d \Omega} \) given by the ratio
between laser-assisted cross section and the field-free cross section.

Figure 1b: Scattering geometries (G1) and (G2). \( k_i \), \( k_f \) are the initial and the final momenta of the incident and scattered electron, \( \theta \) is scattering angle, \( \varepsilon \) is the polarization vector of the electric laser field and \( \Delta \) is the momentum transfer. In (G2): \( \varepsilon \) is parallel to \( \Delta \). In (G3): \( \varepsilon \) is orthogonal to \( \Delta \).

We present in the Figs. 2a, 2b and 2c the scattered electron spectra calculated for an exchange number \( l \) photons in the scattering with \(-6 \leq l \leq 6\) in the three collision geometries (G1), (G2) and (G3).

We can see that the laser assisted intensity signal is more important in (G1) than in (G2) and (G3). We explain this result by a destructive interference between the two waves of the bichromatic laser field in the collision geometries (G2) and (G3). Thus we can use (G1) to promote the constructive interference between these two waves and to get more important signal intensity.

On the other hand, as consequence of generalized Bessel functions properties, we recognize a symmetrical laser assisted signals with respect to \( l = 0 \) in G1 and G2 cases.

3.2. Coherent phase control

We present in the Figs. 3a and 3b the scattered electron spectra calculated as function of the scattering angle \( \theta \) for different values of the phase angle \( \varphi \). One can see that the relative phase \( \varphi \) between the two wavefunctions of the bichromatic laser field plays a very important role in the collision. In the case of two-photon absorption and \( \varphi = \pi \) (the two wavefunctions are in opposition of phase) the laser signal is superior to the case \( \varphi = 0 \) (the two waves are in phase). In case of the emission of two photons, the effect is inversed. At the quadrature of the phase the emission or the absorption of photons doesn't influence the assisted laser signal.

One can see that the signal intensity in the collision is controllable by the phase between the two waves of laser field. The monochromatic laser field doesn't have this faculty.

Figure 2: Scattered electron spectra for \(-6 \leq l \leq 6\) in the three collision geometries. The bichromatic laser parameters are \( \hbar \omega = 0.117 \text{eV} \), \( E_1 = 2.710^5 \text{Vcm}^{-1} \), \( I=10^8 \text{ Wcm}^{-2} \) and the relative phase angle between the two components is zero. The scattering angle is \( \theta = 155^\circ \) and the incident electron energy is \( E_i = 9.5 \text{ eV} \). (a) is done in (G1), (b) in (G2) and (c) in (G3) geometry.
We present in the Fig. 5 the laser assisted signal calculated at a scattering angle $\theta = 155^\circ$ and for an exchange number $l$ of photons with $-6 \leq l \leq 6$ in the scattering geometry (G1), respectively for the monochromatic case and for the bichromatic case. One can see that the signal in the latter is superior to that in the former.

We conclude that the scattered laser assisted signal intensity is sensitive to the geometry of the collision in the bichromatic laser field as in the monochromatic field. Using a second laser field component can enhance the signal intensity, moreover the variation of the phase between the two field components can control the diffusion. It is possible to increase or decrease the laser-assisted signal. Especially at the quadrature of the phase ($\phi = \pi/2$) the signal in the presence of the $\text{CO}_2$ monochromatic laser can be reproduced by a $\text{CO}_2$ bichromatic laser field with less intensity what would be a great advantage in laser spectroscopy. We assure that the coherent phase control can give additional information on the scattering event in the experiments.
References