S. Hennani, H. Nebdi, A. Belafhal*
Laboratoire de Physique Nucléaire, Atomique et Moléculaire,
Département de Physique, Faculté des Sciences, Université Chouaib Doukkali
B.P. 20, El Jadida 24000, Morocco
* Corresponding author. E-mail: belafhal@gmail.com
Received: 15 January 2012; revised version accepted: 05 March 2012

Abstract
In this work, we will derive, by the use of generalized diffraction formula, the propagation of Modified Bessel-modulated Gaussian beams with quadratic radial dependence (MQBG) in a misaligned optical system. We will study the illumination by a MQBG beam of a misaligned system with a circular thin lens and analyse the influence of different parameters of the considered system on the field intensity.

Keywords: MQBG beams; Misaligned optical system; Circular thin lens.

1. Introduction
In last years, propagation of laser beams through a misaligned first-order optical system attracts the attention of some optical researchers. In many application fields, one can show that the error of adjustment of the optical modules and beam path will bring in misalignment in optical system which can be created also by small perturbations [1-4]. In recent years, an increasing research has been put into creating paraxial light beams carrying finite power and having better features than the Bessel-Gauss beams which are a particular case of Kummer beams and called the Modified Bessel-modulated Gaussian beams with quadratic radial dependence [5-10]. In the following, this beams family will be referred as ‘MQBG’ beams.

To the best of our knowledge, propagation of MQBG beams in a misaligned optical property has not been investigated. In this paper, by the use of the generalized Collins formula, called also the generalized diffraction integral, we will consider the MQBG beams as example to determine the analytical formula for the propagation of this beams family in a misaligned optical system. We will give in detail the influence of different parameters of a misaligned system with a circular thin lens on the field intensity of the considered beams.

2. Propagation of MQBG beams through a misaligned system
Following the works of Zhao et al. [11] and, Chafiq et al. [12], for a beams propagates through a misaligned first order optical system and in cylindrical system, the electric field in the output plane is given by the following generalized diffraction formula

\[
E_i(r, \theta, z) = \frac{ik}{2 \pi B} e^{-ikz} \int_{0}^{2\pi} E_0(r_0, \theta_0) \exp\left(-\frac{ik}{2B} \left[A r_0^2 - 2r r_0 \cos(\theta - \theta_0) + Dr^2 + er_0 \cos \theta_0 + fr_0 \sin \theta_0 + gr \cos \theta + hr \sin \theta \right]\right) r_0 dr_0 d\theta_0
\]

where \( E_0 \) is the incident electric field at the input plane given by the definition of MQBG beams, \( (r_0, \theta_0) \) and \( (r, \theta) \) are the radial and azimuth angle coordinates in the input and output planes, respectively. \( k \) is the wave number with \( k = \frac{2 \pi}{\lambda} \), with \( \lambda \) denotes the wavelength. In Eq (1), ABCD denotes the transfer matrix of optical system from the alignment reference planes \( PR_1 \) and \( PR_2 \).

Fig. 1 shows the arrangement of a misaligned first-order optical system characterized by two fundamental parameters, \( \varepsilon \) and \( \varepsilon' \) which are the transverse offset and tilted angle, respectively. The other parameters are shown in Fig. 1.

By introduction of the following four misaligned matrix elements

\[
\alpha_T = 1 - A, \beta_T = \delta - B, \sigma_T = -C \quad \text{and} \quad \delta_T = 1 - D,
\]

where \( \delta \) denotes the axial distance between the input plane and the output plane, and the misaligned azimuth angle \( \varphi \), we define the other four parameters used in Eq. (1), by

\[
\left\{ \begin{array}{l}
\epsilon_A = 2 \alpha_T \left( \cos \varphi \right) + 2 \beta_T \arctan \left( \frac{\cos \varphi}{\sin \varphi} \right) \tan \epsilon' \left( \sin \varphi \right), \\
\end{array} \right.
\]

and
\[
\tilde{g} = 2(B\gamma_T - D\alpha_T)\left[\frac{\cos \varphi}{\sin \varphi}\right] + 2(B\beta_T - D\beta_T)\arctan\left[\frac{\cos \varphi}{\sin \varphi}\right] \tan e'.
\]

(4)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{misaligned_paraxial_abcd_optical_system}
\caption{Misaligned paraxial ABCD optical system.}
\end{figure}

By inserting the following expression of \( E_0 \)
\[
E_0(r_0, \theta_0) = C e^{-i\theta_0} e^{-i\frac{\varphi}{2\omega(\omega^2)}} \int\limits_{\omega(\omega^2)}^{\omega(\omega^2)} \left[ I_{\frac{1}{2}} \left( \frac{r_0^2}{2\omega^2} \right) - I_{\frac{1}{2}} \left( \frac{r_0^2}{2\omega^2} \right) \right] \, d\theta_0,
\]

(5)

Eq. (1) becomes
\[
E_1(r, \theta, \gamma) = C \left( \frac{k}{2\pi} \right) e^{-i\frac{\varphi}{2B}} e^{-i\frac{\varphi}{2B}} \int\limits_{\theta_0}^{\theta_0} \left( \frac{1}{2\omega(\omega^2)} \right) \left[ I_{\frac{1}{2}} \left( \frac{r_0^2}{2\omega^2} \right) - I_{\frac{1}{2}} \left( \frac{r_0^2}{2\omega^2} \right) \right] F_1(r_0) r_0^2 \, d\theta_0,
\]

(6)

where
\[
F_1(r_0) = \int\limits_{0}^{\frac{2\pi}{2}} e^{-i\theta_0} e^{i\frac{\varphi}{2B}} (2r^2 \cos \theta + 2r^2 \sin \theta) \sin \theta_0 \, d\theta_0,
\]

(7)

with the parameters
\[
r_1 = \left[ (2r \cos \theta - \varphi)^2 + (2r \sin \theta - \varphi)^2 \right],
\]

(8)

\[
\cos \phi = \frac{2r \cos \theta - \varphi}{n},
\]

(9)

and
\[
\sin \phi = \frac{2r \sin \theta - \varphi}{n}.
\]

(10)

We know that the azimuthal integration over \( \theta_0 \) yields an \( l \)-th order Bessel function of the first kind [13] and consequently, the quantity \( F_1(r_0) \) can be written as
\[
F_1(r_0) = 2\pi J_{\frac{l}{2}} \left( \frac{kr_0}{2B} \right) \exp \left( i\frac{l}{2} - \phi \right),
\]

(11)

So, Eq. (6) becomes
\[
E(r, \theta, \gamma) = \xi \left( G_{\frac{l}{2}} - G_{\frac{l}{2}} \right),
\]

(12)

where
\[
G_{\frac{l}{2}} = \int\limits_{0}^{\infty} r^2 \exp \left[ -\left( \frac{1}{2\omega^2} + i\frac{kA}{2B} \right) r^2 \right] J_{\frac{1}{2}} \left( \frac{kr_0}{2B} \right) \left( \frac{r^2}{2\omega^2} \right) \, dr,
\]

(13)

\[
G_{\frac{l}{2}} = \int\limits_{0}^{\infty} r^2 \exp \left[ -\left( \frac{1}{2\omega^2} + i\frac{kA}{2B} \right) r^2 \right] J_{\frac{1}{2}} \left( \frac{kr_0}{2B} \right) \left( \frac{r^2}{2\omega^2} \right) \, dr,
\]

(14)

and
\[
\xi = iC \left( \frac{k}{B} \right) e^{-i\varphi} e^{i\frac{\varphi}{2}} \exp \left[ -\left( \frac{k}{2B} \right) (D \varphi^2 + 2r \cos \theta + 2r \sin \theta) \right].
\]

(15)

Its well known that the integrals (13) and (14) can be evaluated by the use of the following function [14]
\[
\Psi_1(a, b; c; w, z) = \sum\limits_{k=0}^{\infty} \sum\limits_{j=0}^{\infty} \frac{(a)_k (b)_j}{(c)_j} \frac{w^k z^j}{k! j!},
\]

(16)
called the Humbert function. We use in this
definition the notation \( (a)_n \) which represents the
Pochhammer symbol defined by \( (a)_n = \frac{\Gamma(a + n)}{\Gamma(a)} \).

By the use of the results of Ref. [14], one can
write for Eqs. (13) and (14)

\[
G_{l_1} = \left( \frac{1}{2} \right) \frac{1}{\omega_{0}^{2} + kA} \left( \frac{k \eta}{4B} \right)^{l_1} \Psi_{l_1} \left( l + 1, \frac{l_2}{2}, l + 1; \xi_1, \xi_2 \right),
\]

where \( \xi_1 = \frac{2B}{2B + ikA\omega_{0}^2} \) and \( \xi_2 = -\frac{k^2 \omega_0^2 \eta^2}{8B(2B + ikA\omega_0^2)} \).

Finally, we find for the output field the following
expression

\[
E_i(r, \theta, \gamma) = iC e^{-ik\Delta} e^{i(kr - \phi)} \exp \left[ -i \frac{k}{2B} (D r^2 + g r \cos \theta + h r \sin \theta) \right] \frac{1}{(l + 1)^{l_1 + 1} \lambda_{0}^{\frac{1}{2}}} \Psi_{l_1} \left( l + 1, \frac{l_2}{2}, l + 1; \xi_1, \xi_2 \right) B \Psi_{l_2} \left( l + 2, \frac{l_2}{2} + 1, l + 1; \xi_1, \xi_2 \right). \tag{19}
\]

This is the general formula of the three-
dimensional intensity of MQBG beams
propagating through a misaligned first order
optical system. This equation is the mean result
of the present work. In the following, we apply
this result in a circular thin lens case.

3. Numerical simulations and discussions
We consider as misaligned system a circular
thin lens illuminated by MQBG beam, the lens has
a focus length \( f' \) and located as \( z = 0 \). Note that

\[
E_i(r, \theta, \gamma) = iC e^{-ikz} e^{i(kr - \phi)} \exp \left[ -i \frac{k}{2z} \right] \frac{1}{2} \left( \frac{1}{l + 1} \right) \lambda_{0}^{\frac{1}{2}} \left( \frac{2z + ik(1 - \frac{z}{f'}, \omega_{0}^2) }{2} \right)^{l_1} \Psi_{l_1} \left( l + 1, \frac{l_2}{2}, l + 1; \xi_1^{\text{lens}}, \xi_2^{\text{lens}} \right) B \Psi_{l_2} \left( l + 2, \frac{l_2}{2} + 1, l + 1; \xi_1^{\text{lens}}, \xi_2^{\text{lens}} \right). \tag{21}
\]

where \( \xi_1^{\text{lens}} = \frac{2z}{2z + ik(1 - \frac{z}{f'}) \omega_0^2} \)

and

\[
E_i(r, \theta, \gamma) = iC e^{-ikz} e^{i(kr - \phi)} \exp \left[ -i \frac{k}{2z} \right] \frac{1}{2} \left( \frac{1}{l + 1} \right) \lambda_{0}^{\frac{1}{2}} \left( \frac{2z + ik(1 - \frac{z}{f'}) \omega_{0}^2} {2} \right)^{l_1} \Psi_{l_1} \left( l + 1, \frac{l_2}{2}, l + 1; \xi_1^{\text{lens}}, \xi_2^{\text{lens}} \right) B \Psi_{l_2} \left( l + 2, \frac{l_2}{2} + 1, l + 1; \xi_1^{\text{lens}}, \xi_2^{\text{lens}} \right). \tag{22}
\]

with...
The intensity of the output field for the following parameters: \( \varepsilon_x = 0.1 \text{mm}, f' = 0.1 \text{mm}, \lambda = 632 \text{mm} \) at the plane \( z = 500 \text{mm} \), 1000mm and 1300mm for different values of the topological charges of MQBG beams.

To illustrate the analytical results, we present, by using Eq. (21), in Figs. 2 and 3 the behavior of the intensity of the output field for the following parameters: \( \varepsilon_x = 0.1 \text{mm}, f' = 0.1 \text{mm}, \lambda = 632 \text{mm} \) at the plane \( z = 500 \text{mm}, 1000 \text{mm} \) and 1300mm for different values of the topological charges of MQBG beams.

\[
\varepsilon_2 = 0 = \frac{k^2 \omega_0^2 \left[ \left( 2r \cos \theta \right)^2 + \left( 2r \sin \theta - f \right)^2 \right]}{8z \left( 2z + ik(1 - z/f)\omega_0^2 \right)}
\]

Figs. 2 and 3 show the evaluation of the intensity distribution at the output plane of the MQBG beams with three values of \( l = 0 \) and 1. One can see that the center of the beam is located on the optical axis in the input plane. One sees from these figures how the situation changes as the beam propagates through the misaligned system. On the other hand, the center of the output beam is shifted from the optical axis by an amount of \( \varepsilon_2 \) in \( x \)-direction and we show how this displacement depends on the topological charge of the beam.

**Figure 2:** Three dimensional intensity distribution and its contour map of the output MQBG beams (online colour). The parameters used are \( \varepsilon_x = 1 \text{mm}, \varepsilon_y = 0, \omega_0 = 0.1 \text{mm}, l = 0 \) at the plane \( z = (a) 500 \text{mm}, (b) 1000 \text{mm} \) and (c) 1300mm.
We have used Eq. (22) to study of the three-dimensional intensity of MQBG beams that we present in Fig. 4-5 for two values of the topological charges. From these figures, one sees how the center of the output beam is shifted from the optical axis by an amount of $f/2$ in y-direction. Figs. 4-5 show that, even for very small changes in the misaligned parameters, the position of the output beam will be greatly changed, but the beam shape of the output intensity remains similar to that of the input even in misaligned system.
Figure 4: Three dimensional intensity distribution and its contour map of the output MQBG beams (online colour). The parameters used are $\epsilon_x = 0 \text{mm}$, $\epsilon_y = 1 \text{mm}$, $\omega_0 = 0.1 \text{mm}$, $l = 0$ at the plane $z = (a) 500 \text{mm}, (b) 1000 \text{mm}$ and (c) $1300 \text{mm}$.
Figure 5: Three dimensional intensity distribution and its contour map of the output MQBG beams (online colour). The parameters used are $e_x = 0\, \text{mm}$, $e_y = 1\, \text{mm}$, $\omega_0 = 0.1\, \text{mm}$, $l = 1$ at the plane $z = (a)\, 500\, \text{mm}, (b)\, 1000\, \text{mm}$ and (c) $1300\, \text{mm}$.

4. Conclusion
In this work, we have derived by the use of the generalized diffraction integral the propagation formula of MQBG beams propagating through a first-order misaligned optical system. We have showed by the use of our results that the output MQBG beam, which its center depends on the misaligned parameters, is a decentered MQBG and its shape still unchanged. Our analytical formulae and numerical examples present good tool for practical use.

References